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A Universal Dependence for Thermal Conductivity of Metals and Dilute Alloys

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A universal curve relating the maximum of thermal conductivity and its respective temperature with the residual electrical resistivity has been proposed for metals and dilute alloys. Based on the equation of that curve, a comparative analysis of selected literature data of thermal conductivity of metals, which have residual electrical resistivity in the range $10^{-11} < \rho_0 < 10^{-5}\ \Omega \cdot \mathrm{cm}$, have been performed. Using the data for 33 metals, confirmation of the Wiedemann–Franz law for the impurity component β/T of thermal conductivity was obtained, which means that $\beta_{\mathrm{th}}/\beta_{\mathrm{el}} \sim 1$, where β_{th} and β_{el} are the parameters of the electron-lattice defect interaction obtained from measurements of thermal and electrical conductivity, respectively. Examples of the failure of the Wiedemann–Franz law are also presented, exhibiting the values of $\beta_{\mathrm{th}}/\beta_{\mathrm{el}}$ in the range 0.16 to 25. Measurements of thermal conductivity in the range 2 to 20 K and determination of the residual electrical resistivity for the samples of Cd doped with Zn and quenched were performed, resulting in values $\beta_{\mathrm{th}}/\beta_{\mathrm{el}} \sim 1$.

KEY WORDS: alloys; electrical resistivity, low temperature; metals; thermal conductivity; Wiedemann–Franz law.

1. INTRODUCTION

For the majority of metals and their dilute alloys, in the region of maximum thermal conductivity k, the lattice component k_{l} can be neglected in comparison to the electronic component k_{e}. Detailed analysis of conditions for the validity of $k_{\mathrm{l}} \ll k_{\mathrm{e}}$ is given in Ref. 1.

Assuming that Matthiessen's rule is satisfied for thermal resistivity, the total thermal resistivity W of a metal at low temperatures can be written as follows:

$$W = \frac{1}{k} \cong \frac{1}{k_{\mathrm{e}}} = \frac{\beta}{T} + AT^2 + CT^4 + DT^4 \tag{1}$$

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where β/T is related to *elastic* scattering of electrons on chemical impurities and physical defects; $AT^2 + CT^4$ is related to *inelastic* scattering of electrons by phonons; DT^4 is related to *elastic* scattering of electrons by phonons, and β, A, C, and D are constant factors.

Terms with T^4 are important at relatively high temperatures, and for $T < 1.5T_{\max}$ Eq. (1) can be approximated [2] by

$$W = \frac{\beta}{T} + A_1 T^2 \tag{2}$$

It follows from analysis of available data of $k(T)$ for metals that replacing exponent 2 in the above equation by an exponent $n \neq 2$ allows most often for better fitting of experimental points to Eq. (2). According to Ref. 3, the value $n > 2$ can be ascribed to the fact that the phonon energy spectrum of metals deviates from the Debye model. Therefore [2],

$$W = \frac{\beta}{T} + A_2 T^n \tag{3}$$

Despite the experimental findings that, for relatively impure samples, n can be a function of β [4], it is generally accepted that the value of n is the characteristic parameter of a particular metal and can vary from 2, e.g., for Ga, to 4.5 for Cd [5].

In Ref. 6 the so-called recommended curves of $k(T)$ for metals are presented, generally approximated by Eq. (3), but with the assumption of dependence between the ideal and the impurity components of thermal resistivity (deviation from Matthiessen's rule):

$$W = \frac{\beta}{T} + \alpha' T^n \tag{4}$$

where $\alpha' = \alpha''(\beta/n\alpha'')^{(m-n)(m+1)}$, and m, n, and α'' are constants, characteristic for the metal [2].

2. THE DEPENDENCE $k_{\max}/T_{\max} = f(\rho_0)$

Let us repeat after Cezairliyan [2] the calculation of the value of the quotient $k_{\max}/T_{\max}$ from the condition for the maximum of the function W in Eq. (3). After simple transformations we obtain

$$k_{\max} = \frac{1}{(n+1)} \left(\frac{n^n}{A_2 \beta^n} \right)^{1/(n+1)} \tag{5}$$

$$T_{\max} = \left(\frac{\beta}{n A_2} \right)^{1/(n+1)} \tag{6}$$

Hence,

$$\frac{k_{\max}}{T_{\max}}=\left(\frac{n}{n+1}\right)\frac{1}{\beta} \tag{7}$$

Thus the quotient $k_{\max}/T_{\max}$ is inversely proportional to the parameter β, determining the scattering of electrons on chemical impurities and physical defects, and depends on the value of n, constant for the particular metal. Since we assumed earlier that β is related exclusively to elastic scattering, we may apply the Wiedemann–Franz law to the impurity electronic thermal resistivity (β/T), obtaining

$$\beta=\frac{\rho_0}{L_0} \tag{8}$$

where ρ_0 is the residual electrical resistivity of the metal and L_0 is the ideal Lorenz number, which is $2.45\times10^{-8}\,\mathrm{W}\cdot\Omega\cdot\mathrm{K}^{-2}$.

Let us denote the value of β determined from measurements of thermal conductivity [using, e.g., Eq. (3) or (7)] by β_{th} and the value of β determined from measurements of electrical conductivity [Eq. (8)] by β_{el}. Therefore,

$$\frac{k_{\max}}{T_{\max}}=\frac{\beta_{\mathrm{el}}}{\beta_{\mathrm{th}}}\left(\frac{n}{n+1}\right)\frac{L_0}{\rho_0} \tag{9}$$

Equation (9) should be the universal one for metals and their dilute alloys, relating the maximum of the thermal conductivity and its respective temperature with the residual electrical resistivity. In cases when the Wiedemann–Franz law is valid, $\beta_{\mathrm{el}}=\beta_{\mathrm{th}}$.

Recently, there have been several works published in which (for some samples) departure from the Wiedemann–Franz law is reported for impurity electronic thermal resistivity in indium [7], copper [8, 10], copper and silver [9], copper, silver, and aluminum [11], and tin doped with zinc [12]. In Ref. 13 some investigations of Refs. 8 and 9 were repeated, using the same materials as in Refs. 8 and 9, and no deviation from the Wiedemann–Franz law was found. In the majority of the investigations cited above, where a deviation was found, the samples were mechanically deformed in various ways, and the failure of the Wiedemann–Franz law was explained, generally, by additional, inelastic scattering: electron-vibrating dislocation or electron kink.

In view of these divergent results, the present authors decided to verify the degree of agreement of Eq. (9) using a large number of experimental data. For simplicity, $n=2$ was assumed for all the metals. The values of

k_{max}, T_{max}, and ρ_0 for 33 various metals were taken from the so-called recommended curves $k(T)$ for metals from the data compiled by the Thermophysical Properties Research Center (TPRC) at Purdue University [6]. All the metals (with the exception of those having no distinct maximum of thermal conductivity) with ρ_0 value less than $0.5 \times 10^{-6}\ \Omega \cdot \mathrm{cm}$ were taken into account. The last criterion, proposed by Klemens [1], distinguishes the metals with the lattice component negligibly small in the simplest way. Ruthenium was not considered, too, because TPRC data for the metal are inconsistent with the results of measurements performed by White [22]. Figure 1 presents collection of the data in the form of the relation k_{max}/T_{max} versus $1/\rho_0$. The points marked with an asterisk in Fig. 1 correspond to the following metals: aluminum, beryllium, cadmium, cesium, chromium, cobalt, copper, gallium, gold, indium, iridium, iron, lead, lithium, magnesium, molybdenum, nickel, niobium, osmium, palladium, platinum, potassium, rhenium, rhodium, rubidium, silver, sodium, tantalum, tungsten, thallium, tin, zinc, and zirconium. Metals, which characterize themselves by the anisotropy of thermal and electrical conductivity, were accepted as such. The best fit (least-squares method) of these experimental points in Fig. 1 was obtained for the curve described by the equation

$$y = ax^p \tag{10}$$

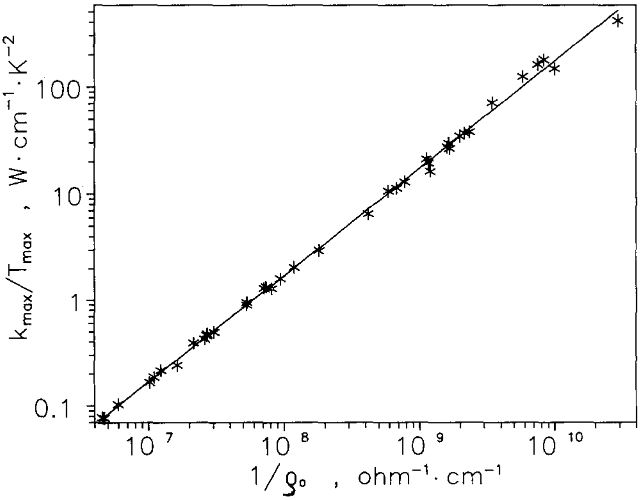

Fig. 1. Dependence of k_{max}/T_{max} on $1/\rho_0$ for 33 metals (listed in text), according to TPRC-recommended curves.

where

$$y = k_{\max}/T_{\max}$$
$$a = 1.5223 \times 10^{-8}\ \mathrm{W \cdot \Omega \cdot K^{-2}}$$
$$x = 1/\rho_0$$
$$p = 1.00615.$$

Obtaining a value of the exponent p very close to unity suggests that the experimental data are well represented by Eq. (9). Approximating the value of p by unity, one obtains the value $\beta_{el}/\beta_{th} = 0.93$. Alternatively, accepting that the obtained deviation of p from 1 is essential, thus adopting $p = 1.00615$, and calculating the integral average value of the function $y(x)/x$ in the range of variation of x from 2×10^6 to $5 \times 10^{10}\Omega^{-1} \cdot \mathrm{cm}^{-1}$ one obtains the averaged value $\beta_{el}/\beta_{th} = 1.08$ for the range of ρ_0 values from 0.5×10^{-6} to $2 \times 10^{-11}\Omega \cdot \mathrm{cm}$. It would be rather difficult to determine exactly the real error of the value of β_{el}/β_{th} thus obtained. It should be remembered that the data for Fig. 1 were obtained in several tens of laboratories across the world over a period of several years. The authors of Ref. 6 have indicated that the error of the TPRC recommended curves

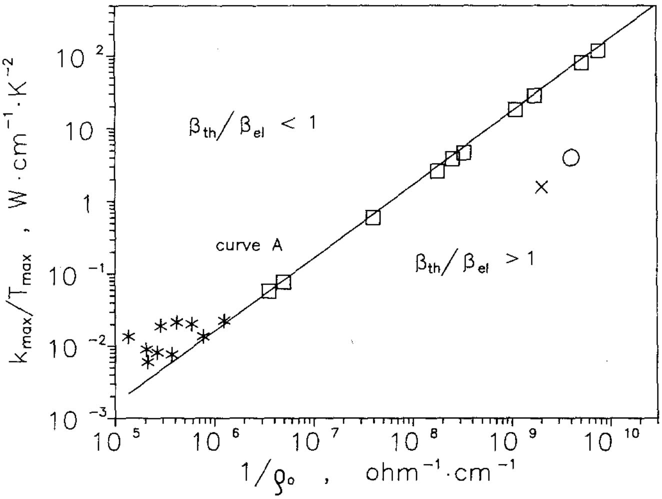

Fig. 2. Dependence of $k_{\max}/T_{\max}$ on $1/\rho_0$ for (□) Al samples of various purity [14], $\beta_{th}/\beta_{el} \cong 1$; (*) RE selected (see text) samples, $\beta_{th}/\beta_{el} \sim 0.17 \rightarrow 0.53$; (○) Cu sample, deformed [8], $\beta_{th}/\beta_{el} \cong 23$; (×) Ag sample, annealed [11], $\beta_{th}/\beta_{el} \cong 25$. Curve A was drawn according to Eq. (10).

(used by us here) may sometimes be as high as 15%; the error of ρ_0 is usually within a few percent.

Considering the above-mentioned facts, the present authors believe that for the TPRC-recommended curves of thermal conductivity of metals, hence for samples of the highest attainable chemical purity and subjected

Table I. The Quotient of the Electron-Lattice Defect Interaction Parameters, Determined from the Thermal and Electrical Conductivity Values: $\beta_{\mathrm{th}}/\beta_{\mathrm{el}}$[a]

Metal	Sample specification	Treatment	ρ_0 ($\Omega \cdot$cm)	$\beta_{\mathrm{th}}/\beta_{\mathrm{el}}$	Ref. No.
Al	Spec. 1	Annealed	1.3×10^{-10}	0.99	14
Al	Spec. 2	Annealed	2.45×10^{-8}	1.06	14
Al	FRW1	Annealed	5.7×10^{-10}	0.99	14
Al	P.H.R.IV	Annealed	2.8×10^{-7}	0.98	14
Sn	99.9%, s.c. ⟨100⟩	Annealed	$\sim 2 \times 10^{-8}$	1.1	18
Sn	99.99%, s.c. ⟨100⟩	Annealed	$\sim 0.8 \times 10^{-8}$	1.1	18
Sn	99.9%, s.c. ⟨111⟩	Annealed	$\sim 2.7 \times 10^{-8}$	1.0	18
Sn	99.99%, s.c. ⟨111⟩	Annealed	$\sim 0.9 \times 10^{-8}$	1.1	18
Sn	99.9%, s.c. ⟨001⟩	Annealed	$\sim 3 \times 10^{-8}$	0.9	18
Sn	99.99%, s.c. ⟨001⟩	Annealed	$\sim 1.2 \times 10^{-8}$	1.0	18
Sn	+0.04% Zn, s.c. ⟨001⟩	Quenched	2.12×10^{-8}	2.0	12
Sn	+0.1% Zn, s.c. ⟨001⟩	Quenched	3.32×10^{-8}	2.9	12
Sn	+0.1% Zn, s.c. ⟨010⟩	Quenched	2.67×10^{-8}	2.1	12
Sn	+0.04% Zn, s.c. ⟨010⟩	Quenched	1.49×10^{-8}	2.0	12
Cu	TPRC r.c.	Annealed	8.51×10^{-10}	0.98	6
Cu	Spec. 1	Deformed	$\rho_{300}/\rho_{4.2} = 6700$	~23	8
Ag	6N	Annealed	8.1×10^{-10}	1.06	17
Ag	6N	Annealed	RRR = 3330	~25	11
Cd	<0.18% Zn, s.c., all[b]				
Cd	Cd 24	Quenched	0.73×10^{-8}	1.0	TW
Cd	Cd 21	Quenched	0.85×10^{-8}	1.0	TW
Cd	Cd 23	Quenched	0.96×10^{-8}	0.8	TW
Cd	Cd 25	Quenched	3.24×10^{-8}	0.9	TW
Ho	TPRC r.c.	Annealed	7.32×10^{-6}	0.17	6
Er	TPRC r.c.	Annealed	3.79×10^{-6}	0.53	6
Gd	TPRC r.c.	Annealed	2.41×10^{-6}	0.31	6

[a] s.c., single crystal; ⟨...⟩, crystallographic direction; TPRC r.c., TPRC-recommended curve; RRR, residual resistances ratio; TW, this work.

[b] The angle between the longitudinal axis of the specimen and the direction ⟨0001⟩, 88°; ⟨−11$\bar{2}$0⟩, 18°; and ⟨10$\bar{1}$0⟩, 12°.

to careful annealing, in the limits of the above-performed analysis, the Wiedemann–Franz law for the β/T component of electronic thermal conductivity is satisfied. Thus, the electron-defect scattering is, in this case, an elastic one.

The question then arises, What are the limits of the applicability of the Wiedemann–Franz law in a metal with varying chemical purity? As an example, aluminum was chosen, for which results of measurements of thermal conductivity for a wide range of ρ_0, from 1.3×10^{-10} to $2.8 \times 10^{-7} \Omega \cdot \text{cm}$, can be found [14]. The results are presented in Fig. 2. Curve A in Fig. 2 was drawn according to Eq. (10). Positioning of the experimental points for Al with respect to the universal curve A allows one to draw the conclusion that the electron-defect scattering is, in this case, an elastic one.

The quotient $\beta_{\text{th}}/\beta_{\text{el}}$ may serve as a sensitive deviation from the Wiedemann–Franz law for the β/T component of thermal conductivity. Table I lists the values of $\beta_{\text{th}}/\beta_{\text{el}}$ for various metals, calculated from Eqs. (7) and (8), on the basis of experimental data from a few, arbitrarily chosen papers. It can be seen that for Al samples of various purity, in the limits of a few percent error in k and ρ_0, values of $\beta_{\text{th}}/\beta_{\text{el}}$ close to unity were obtained, which demonstrates applicability of the Wiedemann–Franz law in this case.

3. DEVIATIONS FROM THE PROPOSED UNIVERSAL DEPENDENCE FOR THE CASE $\beta_{\text{th}}/\beta_{\text{el}} < 1$

Points marked with an asterisk in Fig. 2 were obtained using the thermal conductivity curves recommended by the TPRC and ρ_0 values for the metals, for which $\rho_0 > 0.5 \times 10^{-6} \Omega \cdot \text{cm}$: gadolinium, holmium, erbium, dysprosium, hafnium, lanthanum, lutetium, titanium, thulium, and vanadium [6]. The majority of these elements are the rare earths. The greatest deviations from the recommended curve A are for holium and gadolinium, $\beta_{\text{th}}/\beta_{\text{el}} = 0.17$ and 0.31, respectively (cf. Table I). The curve $k(T)$ used by us for holmium is almost identical to that of Ref. 15. The authors of that paper obtained the value $L_0 = 9.10 \times 10^{-8} \text{W} \cdot \Omega \cdot \text{K}^{-2}$, which, according to their opinion, is the result of not considering the importance in this metal of phonon and magnon components of thermal conductivity. Similarly, the authors of Ref. 16, based on their results of measurements of thermal and electrical conductivity of one gadolinium and two terbium samples, obtained values of L_0 equal to 7.1×10^{-8}, 5.16×10^{-8} and $5.53 \times 10^{-8} \text{W} \cdot \Omega \cdot \text{K}^{-2}$, respectively. In these materials, besides the electron component of thermal conductivity, there are other mechanisms of transport that contribute to the total thermal conductivity of these rare earths [16]. Failing to consider these mechanisms is the reason for positioning the respective points for rare earths above the universal curve A in Fig. 2.

4. DEVIATIONS FROM THE PROPOSED UNIVERSAL DEPENDENCE FOR THE CASE $\beta_{th}/\beta_{el} > 1$

In the case $\beta_{th}/\beta_{el} > 1$, the points obtained from Eq. (9) will fall below the universal curve in Figs. 1 and 2. In Fig. 2, a point (marked with an "X") corresponding to the experimental data of Ref. 11 for one sample of Ag (purity 6N, annealed) and another point (marked with an "O") corresponding to the experimental data of Ref. 8 for one sample of Cu (mechanically deformed) were included as an example. Our estimation of the values of k_{max}/T_{max} and ρ_0 for the two points was accomplished exploiting the characteristics of k_{el} (determined from the Wiedemann–Franz law) presented in Refs. 11 and 8 and k versus temperature and Eqs. (7) and (8) of the present work. In the both cases, a strong deviation of the respective points from curve A is visible, exceeding several times the error of estimation of k_{max}/T_{max} and ρ_0 values. We estimated the value of β_{th}/β_{el} for the first of these samples (Ag, annealed) to be about 25; that for the second (Cu, deformed), about 23 (cf. Table I). In both cases, according to the authors of the respective works, the lack of fulfillment of the Wiedemann–Franz law is supposed to be connected, speaking generally, with the inelastic interaction of electrons with lattice defect, influencing the thermal conductivity more strongly than the electrical conductivity. To the authors of the present work, the fact that this hypothetical effect of inelastic scattering would appear in the 6N-purity, undeformed, annealed sample of Ag is rather surprising. We present in Table I, for comparison, the value $\beta_{th}/\beta_{el} = 1.06$, which we estimated on the basis of the results of Ref. 17, where a 6N-purity, nondeformed, annealed sample of Ag was also investigated.

Let us now compare the results for Sn single-crystal samples of various purity, subjected to the process of either annealing [18] or quenching [12] (Table I). The values of β_{th}/β_{el} for the annealed samples of Sn are close to unity, in the limits of k and ρ determination error. On the other hand, the values of β_{th}/β_{el} for quenched Sn samples, estimated to fall in the range 2.00 to 2.91, exceed strongly the theoretical value 1 and cannot be explained by a few percent error of k and ρ_0 determination. This suggests the possibility of the existence of an additional mechanism of electron scattering, increasing the total thermal resistivity but not influencing essentially the electrical conductivity. Such a mechanism could be provided by inelastic electron-vibrating dislocation scattering or (proposed on theoretical grounds by Mukhin [19]) electron-kink inelastic scattering. The other possibility—inelastic scattering on chemical impurities (found in electrical conductivity of light metals by Koshino [20] in the early sixties)—seems less probable in this case, since for the Sn samples of similar purity, but annealed, it did not exhibit itself [18].

Trying to relate the values of $\beta_{\text{th}}/\beta_{\text{el}} > 1$ obtained for the quenched Sn to the inelastic electron-dislocation scattering, one should consider the following facts.

(a) The quenching of a metal sample may lead to a considerable amount of frozen-in point defects and dislocations which were in thermodynamic equilibrium at the starting temperature of the quenching process.

(b) During the quenching process very strong thermal stresses appear. If these stresses exceed the plastic limit of the metal, the plastic deformation will generate a considerable amount of crystal lattice defects.

(c) The height of k maximum in the quenched Sn sample is remarkably lower than the height of the maximum in samples of a similar degree of purity but subjected to annealing [18].

(d) The majority of works reporting deviations from the Wiedemann–Franz law for the $\beta|T$ component of thermal conductivity concerns mechanically deformed samples. If the plastic limit were really exceeded in Sn in the process of quenching, the plastic deformation would appear in these samples.

5. THERMAL AND ELECTRICAL MEASUREMENTS OF ZINC-DOPED CADMIUM

The hypothesis of the possibility of occurrence of inelastic electron-crystal lattice defect scattering in a quenched metal sample was one of the reasons for which the present authors conducted measurements of thermal and electrical conductivity of quenched samples of Zn-doped cadmium (from 0.007 to 0.18 wt% Zn). Importantly, in the case of slow cooling of the doped metal samples from melting to room temperature, the chemical impurities can group in the region of dislocations, while in the case of quenching one obtains more uniform distribution of the impurities [21].

The thermal conductivity of Cd:Zn was measured by the method of stationary, axial heat flow. The electrical conductivity of the samples was measured potentiometrically, with the aid of a superconducting modulator enabling determination of voltages as low as 10^{-10} V. The values ρ measured in the range of temperatures from 2 to 4.2 K were extrapolated to the value ρ_0 (0 K). Prior to thermal measurements, the samples were

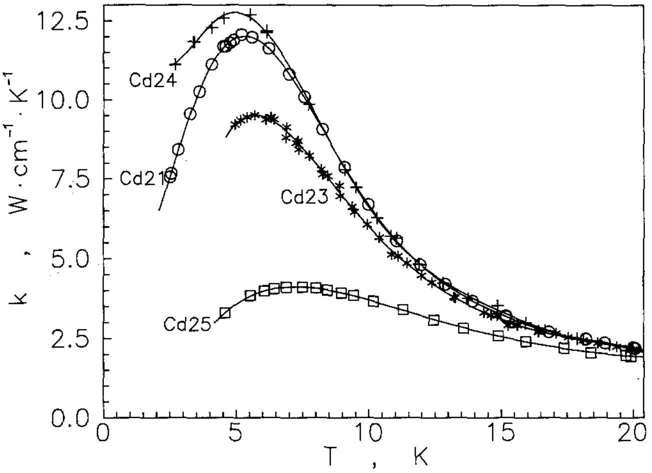

Fig. 3. Temperature dependence of thermal conductivity for Cd:Zn samples, quenched: (□) Cd No. 25, $\rho_0 = 3.24 \times 10^{-8} \Omega \cdot \mathrm{cm}$; (*) Cd No. 23, $\rho_0 = 0.96 \times 10^{-8} \Omega \cdot \mathrm{cm}$; (○) Cd No. 21, $\rho_0 = 0.85 \times 10^{-8} \Omega \cdot \mathrm{cm}$; (+) Cd No. 24, $\rho_0 = 0.73 \times 10^{-8} \Omega \cdot \mathrm{cm}$.

quenched from a temperature close to melting down to room temperature by immersion of the hot container with the samples into LN_2.

In Fig. 3, we present the dependence of thermal conductivity on temperature for four samples of Cd doped with Zn. Table I lists the values of $\beta_{\mathrm{th}}/\beta_{\mathrm{el}}$ for these samples. Within the accuracy limits of k and ρ measurements, it was found that $\beta_{\mathrm{th}}/\beta_{\mathrm{el}}$ is close to unity, which suggests the nonexistence of inelastic electron-crystal lattice defect scattering in the β/T component of the thermal conductivity of the quenched cadmium samples investigated by us.

Assuming that point b in Section 4 may play an essential part in the quenched metal, the different results of measurements for the quenched tin and cadmium may reflect different mechanical properties of these materials (different plastic limits in Sn and Cd).

6. CONCLUSIONS

A universal dependence, given by Eq. (9), relating the thermal and electrical conductivities of metals, is proposed.

Using this relation (for $n = 2$) and on the basis of the 33 TPRC-recommended curves, the averaged value $\beta_{\mathrm{el}}/\beta_{\mathrm{th}} = 0.93$ was obtained. This suggests confirmation of the validity of the assumptions adopted and nonexistence of inelastic electron-lattice defect scattering, in the experimental data analyzed.

The positioning of the experimental results above the universal curve described by the proposed equation, Eq. (9), $\beta_{th}/\beta_{el} < 1$, or below it, $\beta_{th}/\beta_{el} > 1$, means, in both cases, failure of the Wiedemann–Franz law for the β/T component of thermal conductivity of metals. In the first case, there are additional mechanisms increasing thermal conductivity (e.g., in rare earths); in the second, one can expect the existence of additional, inelastic mechanisms of electron scattering on the lattice defects, increasing the thermal resistivity (e.g., in Sn doped with Zn and quenched).

The measurements performed by us of the electrical and thermal conductivity in Cd, doped with Zn and quenched, have shown that there was no inelastic electron-lattice defect scattering in the investigated samples observable within the limits of experimental error of k and ρ determination. It means that the Wiedemann–Franz law for the β/T component of thermal conductivity holds in that case.

7. FINAL REMARKS

The present state of research does not allow one to create a uniform and consistent image of the validity (or failure) of the Wiedemann–Franz law for impurity, electronic thermal resistivity in the case of metals with the electronic thermal conductivity component dominating ($\rho_0 < 0.5 \times 10^{-6} \Omega \cdot \mathrm{cm}$). Considering the relatively great number of works in which the Wiedemann–Franz law has been found to fail, the present authors are rather surprised by the fact that for 33 thermal conductivity curves recommended by the TPRC, the averaged value of β_{el}/β_{th} is close to unity. Is it possible that the relatively small scatter of the points in the Fig. 1 is the result of particular reliability of the TPRC-recommended curves?

Obviously, too little attention has been devoted to the parameters of thermal treatment of samples prior to k and ρ measurements. For example, according to Ref. 17, the change of annealing temperature of 6N Ag samples from 550 to 530°C, in both cases 24^h, caused the value of k_{max} to decrease from $146.6\ \mathrm{W \cdot cm^{-1} \cdot K^{-1}}$ at $T_{max} = 7.14\ \mathrm{K}$ to $137.4\ \mathrm{W \cdot cm^{-1} \cdot K^{-1}}$ at $T_{max} = 7.83\ \mathrm{K}$. Standardization of the thermal treatment parameters of metal samples would allow more meaningful comparison of the results from various laboratories. Presently, unification of the methods of thermal and electrical conductivity determination does not seem possible for various reasons. However, application of Thermal Conductivity and Electrical Conductivity Measurement Standards, advanced in recognized research centers, would increase the reliability and compatibility of the obtained results.

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